Calculus Warm Up #5-3

Locate the extrema on the indicated interval:

1. $f(x) = x^3 - 12x$

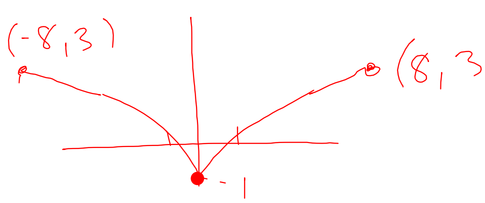
   on $[0, 4]$

2. $g(x) = \sqrt[3]{x}$

   on $[-1, 1]$

9) $f(x) = x^{2/3} - 1 \quad [-8, 8]$

GCD:



HW Questions:

11) $f(x) = \dfrac{x^2 - 2x - 3}{x+2}$ $\qquad [-1, 3]$

$$\begin{array}{r|rrr} -2 & 1 & -2 & -3 \\ & & -2 & \\ \hline & 1 & -4 & \end{array}$$

$y = x - 4$

$f(1) = \dfrac{1 + 2 - 3}{1}$

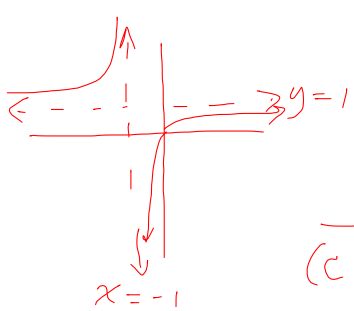
$f(3) =$

$f'(x) = \dfrac{x^2 + 4x - 1}{(x+2)^2}$

17) $f(x) = \dfrac{x}{x+1}$ $\qquad \left[-\frac{1}{2}, 2\right]$

$f\left(-\frac{1}{2}\right) =$

$f(2) =$



$$\frac{1}{(c+1)^2} = \frac{f(2) - f\left(-\frac{1}{2}\right)}{\frac{2}{2} \cdot \frac{2}{1} + \frac{1}{2}}$$

$$\frac{\frac{2}{3} + \frac{3}{3}}{\frac{5}{2}}$$

$f'(x) = \dfrac{1}{(x+1)^2}$

$$\frac{1}{(c+1)^2} = \frac{\cancel{5}}{3} \cdot \frac{2}{\cancel{5}}$$

$$1 = \frac{2(c+1)^2}{3}$$

HW Questions:

19) $f(x) = x^3 \quad [0,1]$

$f'(x) = 3x^2$

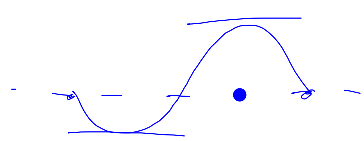
$$3c^2 = \frac{f(1) - f(0)}{1 - 0}$$

$$c^2 = \frac{1}{3}$$

$$c = \pm \frac{\sqrt{3}}{3}$$

$f'(t) = -32t + 48$

$0 = -32t + 48$



$[1,2]$

HW Questions:

23) a) $\text{avg vel.} = \dfrac{s(3) - s(0)}{3 - 0}$

b) $s'(t) = \dfrac{s(3) - s(0)}{3 - 0}$

$-32t = -48$

t

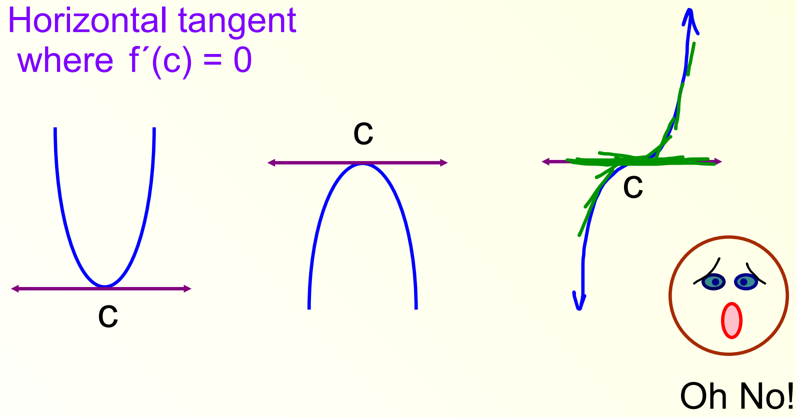
4.3

-Increasing and decreasing functions

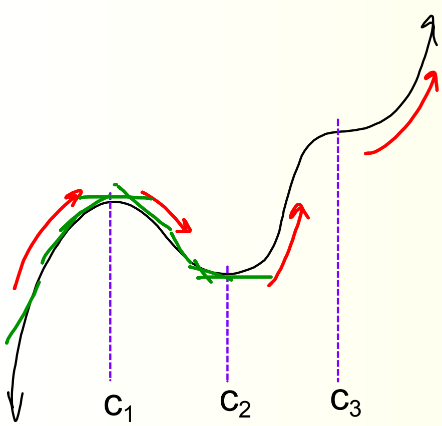
-The First Derivative Test

-Strictly monotonic functions

Limitations of Rolle's Theorem: Looking for <u>possible</u> extrema where $f'(x) = 0$:



Read from left to right: find places where a function changes from increasing to decreasing or decr. to incr.



f is increasing on $(-\infty, c_1)$

f is decreasing on (c_1, c_2)

f is increasing on (c_2, ∞)

Max at $x = c_1$, where f changes from incr. to decr.
Min at $x = c_2$, where f changes from decr. to incr.
No change at c_3, so no extrema there.

Look at f(x) from the perspective of slopes:

THEOREM 4.5
TEST FOR INCREASING OR DECREASING FUNCTIONS

Let f be a function that is differentiable on the interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on (a, b).
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on (a, b).
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on (a, b).

Slopes, f′(x), are positive when f(x) is increasing.

Slopes, f′(x), are negative when f(x) is decreasing.

EXAMPLE 1 Determining intervals on which f is increasing or decreasing

Find the open intervals on which

$$f(x) = x^3 - \frac{3}{2}x^2$$

is increasing or decreasing.

1. Find critical numbers (where f′(x) = 0 or undefined)

2. Test an x value in each region for slopes + or - to see where function changes from incr. to decr. or the other way.

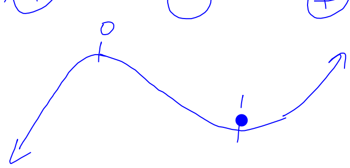
$f'(x) = 3x^2 - 3x$

$0 = 3x(x-1)$

$\boxed{x = 0, 1 \text{ critical \#'s}}$

f′ test:

−1		½		2
− − (+)	0	+ − (−)	1	+ + (+)



f is increasing $(-\infty, 0) \cup (1, \infty)$

f is decreasing $(0, 1)$

THEOREM 4.6
THE FIRST DERIVATIVE TEST

Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1. If f' changes from negative to positive at c, then $f(c)$ is a **relative minimum** of f.
2. If f' changes from positive to negative at c, then $f(c)$ is a **relative maximum** of f.
3. If f' does not change signs at c, then $f(c)$ is neither a relative minimum nor a relative maximum.

EXAMPLE 2 Applying the First Derivative Test

(means test x-values to see where the slopes change sign.)

Use the First Derivative Test to find all relative maxima and minima for the function given by

$$f(x) = 2x^3 - 3x^2 - 36x + 14.$$

$f'(x) = 6x^2 - 6x - 36$

$0 = 6(x^2 - x - 6)$

$0 = 6(x-3)(x+2)$

critical #'s: $-2, 3$

f' test

-3	0	4
+ − −	+ − +	+ + +
⊕	⊖	⊕

−2, 3

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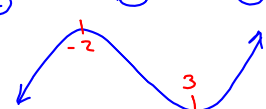

Max @ $x = -2$

Min @ $x = 3$

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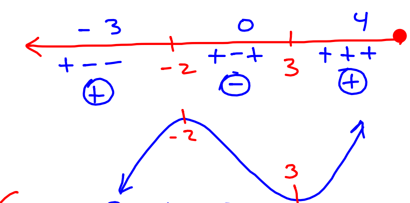


$f(-2) = 58$
$f(3) = -67$ $\leftarrow$ $\begin{cases} \text{Max @ } x = -2 \\ \text{Min @ } x = 3 \end{cases}$

So maximum: $(-2, 58)$ and minimum: $(3, -67)$

You try

Find the relative extrema:

$$f(x) = (x^2 - 4)^{2/3}$$

1. $f'(x)$
2. critical #'s
3. test $\longleftrightarrow$

EXAMPLE 3 Applying the First Derivative Test

Find the relative extrema of $f(x) = (x^2 - 4)^{2/3}$.

① $f'(x)$
② critical #'s
③ test

$$f'(x) = \frac{4x}{3\sqrt[3]{x^2-4}}$$

$x = 0, \pm 2$

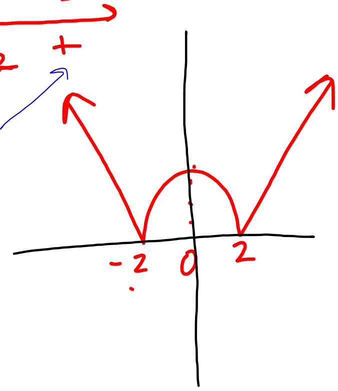
f′ test

−3 −1 1 3

− −2 + 0 − 2 +

min $(\pm 2, 0)$

max $(0, 4^{2/3})$



Use the first derivative test to locate the relative extrema of

$$f(x) = \frac{x^4 + 1}{x^2}$$

EXAMPLE 4 The First Derivative Test and points of discontinuity

Find the relative extrema of $f(x) = (x^4 + 1)/x^2$.

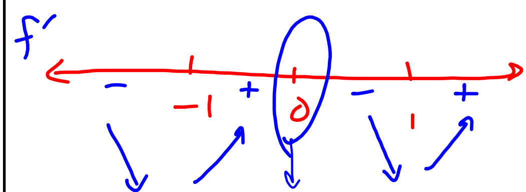
$$f'(x) = \frac{2(x^2+1)(x^2-1)}{x^3}$$

critical #'s

$x = 0, \pm 1$

f' undef. (for $x = 0$)

$f' = 0$ (for $x = \pm 1$)



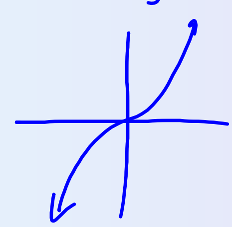
f(0) is undefined also, so no extrema there

Minima: $(-1, 2)$ & $(1, 2)$

A function is strictly monotonic if it is either increasing over the entire domain or decreasing over the entire domain.

The domain could either be implied or on a specified interval.

Ex: $y = x^3$



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